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Blind Quality Estimation for Corrupted Source Signals Based on A-Posteriori Probabilities

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Abstract — A novel approach is presented for assessing the quality of transmission systems, comprising quantized source signals and APP source decoders, via Monte-Carlo simulation. A-posteriori probabilities are exploited in order to obtain an unbiased estimate of both the symbol error probability and the expected distortion for the transmission system; knowledge of the transmitted source signal is not necessary. Compared to the conventional method this blind quality estimation has a smaller estimation variance.

SUMMARY

The bit error rate estimation based on a-posteriori probabilities (APPs) was shown to be superior to the conventional one based on hard decisions [1]. In this paper this method is extended to symbol error rate (SER) and distortion estimation.

Let us assume a simplified transmission model, where a real-valued source signal¹ U is quantized to quantization indices I , $i \in \mathcal{I}$, which are transmitted over a communication channel. Based on the channel observations Y the receiver generates APPs $Pr(I=i|\mathbf{y})$ [2], which are exploited to obtain estimates \hat{U} and \hat{I} of U and I .

Typically quality evaluation via Monte-Carlo simulation is based on a comparison of the transmitted source data (u, i) to their reconstructed versions (\hat{u}, \hat{i}) with respect to appropriate quality measures, such as the symbol error rate P_s or distortion D , defined as $P_s := Pr(I \neq \hat{I})$ and $D := E\{(U - \hat{U})^2\}$. Accordingly the conventional approach for measuring P_s and D can be summarized as follows:

Method H: Let us define the *hard SER sample* $z_H := Pr(I \neq \hat{I} | I = i, \hat{I} = \hat{i})$, $z_H \in [0, 1]$, indicating whether a symbol error occurred or not, and let us define the *hard distortion sample* $d_H := (u - \hat{u})^2$, taking into account the contribution to the reconstruction error due to estimate \hat{u} . For a transmission of K source symbols, the corresponding quality samples can be used to compute the *hard SER estimate* $z_H^{(K)}$ and *hard distortion estimate* $d_H^{(K)}$ as

$$z_H^{(K)} := \frac{1}{K} \sum_{k=1}^K z_{Hk} \quad \text{and} \quad d_H^{(K)} := \frac{1}{K} \sum_{k=1}^K d_{Hk}.$$

Obviously, $z_H^{(K)}$ and $d_H^{(K)}$ rely on the knowledge of u and i , from which it follows that the conventional Method H is not suitable for application in practical transmission systems.

Thus, we consider now the case, where knowledge of u and i is not available. I.e., only the source statistics, the estimates \hat{u} and \hat{i} , and the set of APPs $p_{A_k} = \{Pr(I_k = i|\mathbf{y}) | i \in \mathcal{I}\}$ may be used for quality estimation. These restrictions lead to a novel approach for the evaluation of P_s and D , referred to as *Method S* in the following:

Method S: We define the *soft SER sample* as $z_S := Pr(I \neq \hat{I} | I = \hat{i}, P_A = p_A)$, which can be computed as $z_S = 1 - Pr(I = \hat{i}|\mathbf{y})$, and we define the *soft distortion sample* $d_S := E\{(U - \hat{U})^2 | P_A = p_A\}$, which is given by the *a-posteriori* expectation of the mean-squared error according to $d_S = \sum_{i \in \mathcal{I}} E\{(u - \hat{u})^2 | I = i\} \cdot Pr(I = i|\mathbf{y})$ for

a given \hat{u} . Considering again the transmission of K source symbols, the *soft SER estimate* $z_S^{(K)}$ and the *soft distortion estimate* $d_S^{(K)}$ for Method S are given by

$$z_S^{(K)} := \frac{1}{K} \sum_{k=1}^K z_{Sk} \quad \text{and} \quad d_S^{(K)} := \frac{1}{K} \sum_{k=1}^K d_{Sk}.$$

For comparison of both methods we regard the hard and the soft SER and distortion samples as random variables Z_H , Z_S and D_H , D_S . From their definitions and since the estimates are sample means, it follows that $\mu_Z = E\{Z_H\} = E\{Z_S\} = P_s$ and $\mu_D = E\{D_H\} = E\{D_S\} = D$. Thus, both estimates are unbiased for both the SER and the distortion estimation.

An appropriate figure-of-merit is the estimation variance. The variance of the hard SER sample Z_H can be written as

$$\sigma_{Z_H}^2 = E\{Z_H^2\} - \mu_Z^2 = E\{Z_H\} - \mu_Z^2 = \mu_Z(1 - \mu_Z), \quad (1)$$

where the identity $Z_H^2 = Z_H$ was applied. The variance of the soft SER sample Z_S , respectively, can be written as

$$\sigma_{Z_S}^2 = E\{Z_S^2\} - \mu_Z^2 \leq E\{Z_S\} - \mu_Z^2 = \mu_Z(1 - \mu_Z) \quad (2)$$

and is upper bounded by $\mu_Z(1 - \mu_Z)$, since from $Z_S \in [0, 1]$ it follows that $Z_S^2 \leq Z_S$, and thus $E\{Z_S^2\} \leq E\{Z_S\}$. Equality holds for the uninteresting cases $Z_S=0$ and $Z_S=1$ ($\sigma_{Z_S}^2=0$). For all other cases we have a lower bound on the ratio of variances $\sigma_{Z_H}^2$ and $\sigma_{Z_S}^2$ of the SER samples:

$$\frac{\sigma_{Z_H}^2}{\sigma_{Z_S}^2} > 1, \quad (3)$$

resulting directly from (1) and (2).

A similar bound on the ratio of variances $\sigma_{D_H}^2$ and $\sigma_{D_S}^2$ of the distortion samples can be derived by applying Jensen's inequality to the a-posteriori expectation of D_H^2 :

$$E\{D_H^2 | P_A = p_A\} \geq E\{D_H | P_A = p_A\}^2 = D_S^2, \quad (4)$$

where the identity $D_H = (u - \hat{u})^2$ and the definition of the soft distortion sample D_S is exploited. It follows from (4) that $E\{D_H^2\} \geq E\{D_S^2\}$, where again equality holds for $\sigma_{D_S}^2 = 0$ (see above), and otherwise

$$\frac{\sigma_{D_H}^2}{\sigma_{D_S}^2} > 1, \quad (5)$$

which represents a lower bound on the ratio of variances $\sigma_{D_H}^2$ and $\sigma_{D_S}^2$ of the distortion samples.

The bounds in (3) and (5) prove that the hard SER sample as well as the hard distortion sample have always (except for $P_s = 0$) a larger variance than the soft SER sample and the soft distortion sample, respectively. This reveals the superiority of the proposed Method S to the conventional Method H. In numerical results for Gauss-Markov sources the gain with respect to the estimation variance turned out to be even larger than predicted.

REFERENCES

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¹Random variables are denoted by uppercase letters, their realizations by lowercase letters. Indices are omitted for convenience, whenever this can be done without ambiguity.